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An extension of the 16th Hilbert problem for continuous piecewise linear–quadratic centers separated by a non-regular line

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ABSTRACT

In the last few decades, there has been much interest in studying piecewise differential systems. This is mainly due to the fact that these differential systems allow us to modelize many natural phenomena. In order to describe the dynamics of a differential system, we need to control its periodic orbits and, especially, its limit cycles. In particular, providing an upper bound for the maximum number of limit cycles that such differential systems can exhibit would be desirable, that is solving the extended 16th Hilbert problem. In general, this is an unsolved problem. In this paper, we give an upper bound for the maximum number of limit cycles that a class of continuous piecewise differential systems formed by an arbitrary linear center and an arbitrary quadratic center separated by a non-regular line can exhibit. So for this class of continuous piecewise differential systems, we have solved the extended 16th Hilbert problem, and the upper bound found is seven. The question whether this upper bound is sharp remains open.

Solving the 16th Hilbert problem, that is, to give an upper bound for the maximum number of limit cycles that a given family of differential systems can exhibit, is, in general, an open problem. In this paper, we obtain a solution to the 16th Hilbert problem for the class of continuous piecewise differential systems formed by a linear and a quadratic system, both having a center, whose continuity manifold is a non-regular line. The methodology includes the use of first integrals and Chebyschev's theory.

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULT

In the last 20 years, there has been much interest in understanding piecewise differential systems mainly due to their relevant applications in modeling many different natural phenomena; see, for instance, the books of Refs. 1–3 and 30, the survey of Ref. 4, and the references cited in these books and in the survey.

In order to describe the dynamics of a differential system, the periodic orbits play a main role (specially the limit cycles, i.e., the periodic orbits isolated in the set of all periodic orbits). Examples of relevant applications of the existence of limit cycles in the dynamics can be found in Refs. 2 and 5–7.

The existence and the number of limit cycles of distinct classes of piecewise differential systems have been studied for many authors, see, for instance, Refs. 8–23, 31, and 32 the list being not exhaustive.

In this work, we consider continuous piecewise differential systems of the form

$$(\dot{x},\dot{y}) = \begin{cases} \mathbf{F}_1(x,y) = (f_1(x,y), f_2(x,y)) \text{ if } (x,y) \in \mathcal{R}_1, \\ \mathbf{F}_2(x,y) = (g_1(x,y), g_2(x,y)) \text{ if } (x,y) \in \mathcal{R}_2, \end{cases} \quad (1)$$

where the dot means derivative in the variable t, and f_i and g_i for $i = 1, 2$ are, respectively, linear and quadratic polynomials. The