QUADRATIC DIFFERENTIAL SYSTEMS WITH A FINITE SADDLE–NODE AND AN INFINITE SADDLE–NODE $(1,1)SN$ - (B)

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Our goal is to make a global study of the class $\mathbf{QsnSN_{11}}$ of all real quadratic polynomial differential systems which have a finite semi–elemental saddle–node and an infinite saddle–node formed by the coalescence of a finite and an infinite singularities. This class can be divided into two different families, namely, $\mathbf{QsnSN_{11}(A)}$ phase portraits possessing a finite saddle–node as the only finite singularity and $\mathbf{QsnSN_{11}(B)}$ phase portraits possessing a finite saddle–node and also a simple finite elemental singularity. Each one of these two families is given by a specific normal form. The study of family $\mathbf{QsnSN_{11}(A)}$ was done in [Artés *et al.*, 2020b] where the authors obtained 36 topologically distinct phase portraits for systems in the closure $\overline{\mathbf{QsnSN_{11}(A)}}$. In this paper we provide the complete study of the geometry of family $\mathbf{QsnSN_{11}(B)}$. This family modulo the action of the affine group and time homotheties is three–dimensional and we give the bifurcation diagram of its closure with respect to a specific normal form, in the three–dimensional real projective space. The respective bifurcation diagram yields 631 subsets with 226 topologically distinct phase portraits for systems in the closure $\overline{\mathbf{QsnSN_{11}(B)}}$ within the representatives of $\mathbf{QsnSN_{11}(B)}$ given by a specific normal form. Some of these phase portraits can be proven to have at least 3 limit cycles.
Keywords: Quadratic differential systems; finite saddle–node; finite elemental singularity; infinite saddle–node; phase portraits; bifurcation diagram; algebraic invariants.