

# QUADRATIC DIFFERENTIAL SYSTEMS WITH A FINITE SADDLE–NODE AND AN INFINITE SADDLE–NODE $(1, 1)SN - (A)$

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Our goal is to make a global study of the class  $\mathbf{QsnSN}_{11}$  of all real quadratic polynomial differential systems which have a finite semi–elemental saddle–node and an infinite saddle–node formed by the coalescence of a finite and an infinite singularities. This class can be divided into two different families, being (A) possessing the finite saddle–node as the only finite singularity and (B) possessing the finite saddle–node and also a finite simple elemental singularity. In this paper we provide the complete study of the geometry of family (A). The family (A) modulo the action of the affine group and time homotheties is four–dimensional and we give the bifurcation diagram of its closure with respect to a specific normal form, in the four–dimensional real projective space  $\mathbb{RP}^4$ . As far as we know, this is the first time that a complete family is studied in the four–dimensional real projective space. The respective bifurcation diagram yields 36 topologically distinct phase portraits for systems in the closure  $\overline{\mathbf{QsnSN}_{11}(\mathbf{A})}$  within the representatives of  $\mathbf{QsnSN}_{11}(\mathbf{A})$  given by a specific normal form.

*Keywords:* Quadratic differential systems; finite saddle–node; infinite saddle–node; phase portraits; bifurcation diagram; algebraic invariants.