RESONANCE OF BOUNDED ISOCHRONOUS OSCILLATORS

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Abstract. An oscillator is called isochronous if all motions have a common period. When the system is forced by a time-dependent perturbation with the same period the phenomenon of resonance may appear. We give a sufficient condition on the perturbation in order that resonance occurs when the period annulus of the isochronous oscillator is bounded. In this context, resonance means that all solutions escape from the period annulus.

1. Introduction

An oscillator with equation

$$\ddot{x} + V'(x) = 0 \tag{1}$$

is called isochronous if it only has one equilibrium point and all solutions in a neighbourhood are periodic with a fixed period, lets say $T = 2\pi$. When a small periodic perturbation with the same period as the isochronous center is added to the force, the phenomenon of resonance may occur. That is, all solutions of the non-autonomous equation

$$\ddot{x} + V'(x) = \varepsilon p(t) \tag{2}$$

are unbounded for $\varepsilon \neq 0$ small. In the recent years the classical theory of resonance has been extended from the linear oscillator to nonlinear isochronous oscillators. We refer for instance [3, 7] for the construction of forcings and [11] for sufficient conditions to produce resonance. Also [2, 5] where the authors treated the specific case of the asymmetric oscillator.

Until now the oscillators treated have been defined over the whole real line or they have had an asymptote. In the first case the potential generates a global center in $\mathbb{R}^2$ whereas in the second case the center is defined in a semi-plane. In both situations the center is global. That is, all solutions are well-defined and 2π-periodic. In this work we treat the case when the isochronous oscillator is bounded (see Figure 1.) In general, a planar bounded center is usually confined inside a homoclinic or heteroclinic connection. In particular, an equilibrium of the equation (1) can be found at the outer boundary of the period annulus. Clearly, this situation is incompatible with isochronicity. However, bounded isochronous centers can be constructed using a singular potential function. In order to differ from the second case mentioned above, the singularity must be integrable. That is, equation (1) is singular but the Hamiltonian $H(x, \dot{x}) = \frac{1}{2}\dot{x}^2 + V(x)$ is not (see [4, 13] and references therein.) If this is the case, equation (1) is said to have a weak singularity. Our main result shows that the resonance condition given in [11] for global centers also produces resonance for the bounded isochronous oscillator. The main difference is that in the present situation resonance is understood as the escape from the period annulus. More precisely, for a given non-empty compact subset $\mathcal{K}$ inside the bounded period

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