Asymptotic expansion of the Dulac map and time for unfoldings of hyperbolic saddles: local setting

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Abstract. In this paper we study unfoldings of planar vector fields in a neighbourhood of a hyperbolic resonant saddle. We give a structure theorem for the asymptotic expansion of the local Dulac time (as well as the local Dulac map) with the remainder uniformly flat with respect to the unfolding parameters. Here local means close enough to the saddle in order that the normalizing coordinates provided by a suitable normal form can be used. The principal part of the asymptotic expansion is given in a monomial scale containing a deformation of the logarithm, the so-called Roussarie-Ecalle compensator. Especial attention is paid to the remainder's properties concerning the derivation with respect to the unfolding parameters.

Contents

1 Introduction and statements of the results

In this paper we study unfoldings of planar vector fields in a neighbourhood of a hyperbolic resonant saddle. It can be viewed as the continuation of a previous paper where we give a $\mathscr{C}^K$ normal form for the unfolding with respect to the conjugacy relation, see [10, Theorem A]. By means of this normal form in that paper