

Asymptotic expansion of the Dulac map and time for unfoldings of hyperbolic saddles: local setting

D. Marín and J. Villadelprat

*BGSMath and Departament de Matemàtiques, Facultat de Ciències,
Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

*Departament d'Enginyeria Informàtica i Matemàtiques, ETSE,
Universitat Rovira i Virgili, 43007 Tarragona, Spain*

Abstract. In this paper we study unfoldings of planar vector fields in a neighbourhood of a hyperbolic resonant saddle. We give a structure theorem for the asymptotic expansion of the local Dulac time (as well as the local Dulac map) with the remainder uniformly flat with respect to the unfolding parameters. Here local means close enough to the saddle in order that the normalizing coordinates provided by a suitable normal form can be used. The principal part of the asymptotic expansion is given in a monomial scale containing a deformation of the logarithm, the so-called Roussarie-Ecalle compensator. Especial attention is paid to the remainder's properties concerning the derivation with respect to the unfolding parameters.

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1 Introduction and statements of the results

In this paper we study unfoldings of planar vector fields in a neighbourhood of a hyperbolic resonant saddle. It can be viewed as the continuation of a previous paper where we give a \mathcal{C}^K normal form for the unfolding with respect to the conjugacy relation, see [10, Theorem A]. By means of this normal form in that paper

2010 *AMS Subject Classification*: 34C07; 34C20; 34C23.

Key words and phrases: Dulac map, Dulac time, asymptotic expansion, uniform flatness.

This work has been partially funded by the Ministry of Science, Innovation and Universities of Spain through the grants MTM2015-66165-P and MTM2017-86795-C3-2-P, the Agency for Management of University and Research Grants of Catalonia through the grants 2017SGR1725 and 2017SGR1617, and by the "María de Maeztu" Programme for Units of Excellence in R&D (MDM-2014-0445).