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DYNAMICS OF THE FITZHUGH-NAGUMO SYSTEM HAVING INVARIANT ALGEBRAIC SURFACES

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ABSTRACT. In this paper we study the dynamics of the FitzHugh-Nagumo system $\dot{x} = z,\ \dot{y} = b(x - dy),\ \dot{z} = x(x-1)(x-a) + y + cz$ having invariant algebraic surfaces. This system has four different types of invariant algebraic surfaces. The dynamics of the FitzHugh-Nagumo system having two of these classes of invariant algebraic surfaces have been characterized in [21]. Using the quasi-homogeneous directional blow up and the Poincaré compactification, we describe the dynamics of the FitzHugh-Nagumo system having the two remaining classes of invariant algebraic surfaces. Moreover for these FitzHugh-Nagumo systems we prove that they do not have limit cycles.

1. INTRODUCTION

The FitzHugh-Nagumo system is given by the partical differential system

$$u_t = u_{xx} - f(u) - v, \quad v_t = \varepsilon(u - \gamma v), \tag{1}$$

where $f(u) = u(u-1)(u-a)$, and $0 < a < 1/2$, $\varepsilon > 0$, $\gamma > 0$ are parameters. We say that a bounded solution $(u,v)(x,t)$ of the FitzHugh-Nagumo system (1) with $x, t \in \mathbb{R}$ is a *travelling wave* if $(u,v)(x,t) = (u,v)(\xi)$, where $\xi = x + ct$ and c is the constant denoting the wave speed. Substituting $u = u(\xi), v = v(\xi)$ into (1), we obtain the ordinary differential system

$$\begin{aligned} \dot{x} &= z = P(x,y,z), \\ \dot{y} &= b(x - dy) = Q(x,y,z), \\ \dot{z} &= x(x-1)(x-a) + y + cz = R(x,y,z). \end{aligned} \tag{2}$$

Here the dot denotes derivative with respect to ξ, $x = u$, $y = v$, $z = \dot{u}$, $b = \varepsilon/c$ and $d = \gamma$, see for more details [11].

The FitzHugh-Nagumo system (1) is classical differential system introduced independently by FitzHugh [8] and Nagumo et al. [19]. It is an important model for describing the excitation of neural membranes and the propagation of nerve impulses along an axon. Besides its biological interest, the FitzHugh-Nagumo system has gained wide investigation from the mathematical point of view, such as the existence, uniqueness and stability of its traveling wave solutions, see for instance [2, 9, 12–14, 19], etc.

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