ENTROPY AND PERIODS FOR CONTINUOUS GRAPH MAPS

JAUME LLIBRE[1] AND VÍCTOR F. SIRVENT[2]

Abstract. For continuous maps from a topological graph into itself we provide new relationships between their topological entropy, their homology and their periods.

1. Introduction and statement of the main results

A *topological graph* or simpliy a *graph* G is a compact connected space having a finite set of points V such that $G \setminus V$ consists of finitely many connected components each of them homeomorphic to an open interval. Some graphs are homotopic to particular cases of wedge sums of circles, which we shall define later on. However not all graphs can be obtained in this way, e.g. the interval, in general any tree, the topological space with the shape of the capital letter sigma. We say that a *graph is trivial* if it is homotopic to a circle or to a point, e.g. intervals and trees are trivial graphs.

Given topological spaces X and Y with chosen points $x_0 \in X$ and $y_0 \in Y$, then the *wedge sum* $X \vee Y$ is the quotient of the disjoint union X and Y obtained by identifying x_0 and y_0 to a single point (for details, see [9, pp. 10]). The wedge sum is also known as "one point union". For example, $\mathbb{S}^1 \vee \mathbb{S}^1$ is homeomorphic to the figure of shpe "8", two circles touching at a point. Some graphs can be obtained as particular cases of wedge sums of $\mathbb{S}^1$, and a compact connected graph X such that $\dim(H_1(X,\mathbb{Q})) = s$ is homotopic to $\mathbb{S}^1 \vee \overset{s-\text{times}}{\cdots} \vee \mathbb{S}^1$, as usual here $H_1(X,\mathbb{Q})$ denotes the first homology group of the topological space X with coefficients in $\mathbb{Q}$. These spaces are also called *bouquet of circles*, we denote by $G_s := \mathbb{S}^1 \vee \overset{s-\text{times}}{\cdots} \vee \mathbb{S}^1$.

Since our techniques rely on homology, we shall mainly consider bouquet of circles, i.e. graphs of the type G_s, for some integer $s > 1$.

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