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Periodic structure of transversal maps on sum-free products of spheres

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ABSTRACT

In this article, we study the periodic structure of transversal maps on the product of spheres of different dimensions. In particular, we give sufficient conditions in order that such maps have infinitely many even and odd periods. Moreover, we also provide sufficient conditions for having non-zero Lefschetz numbers of period *m* for infinitely many *m*'s. We extend these results to transversal maps on rational exterior spaces of rank 1.

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1. Introduction and statement of the main results

Let f be a continuous self-map on X. If $x \in X$ and $f(x) = x$, we say that x is a *fixed point* of the map f. If $f^n(x) = x$ and $f^k(x) \neq x$ for all $k = 1, \dots, n-1$, then we say that x is a *periodic point* of the map f of *period* n. We denote by Per (f) the set of the periods of all periodic points of a map $f : X \to X$.

Let X be a n-dimensional topological manifold and f a continuous self-map on X. The map f induces a homomorphism on the kth rational homology group of X for $0 \leq k \leq n$, i.e. $f_{*k} : H_k(X, \mathbb{Q}) \to H_k(X, \mathbb{Q})$. The $H_k(X, \mathbb{Q})$ is a finite dimensional vector space over $\mathbb{Q}$ and f_{*k} is a linear map whose matrix has integer entries.

The *Lefschetz number* of the map f is an integer defined as

$$L(f) = \sum_{k=0}^{n} (-1)^k \text{trace}(f_{*k}).$$

The *Lefschetz Fixed Point Theorem* states that if $L(f) \neq 0$ then f has a fixed point (cf. [2] or [15]).

The *Lefschetz numbers of period m* are defined by

$$\ell(f^m) := \sum_{r|m} \mu(r) L(f^{m/r}),$$

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