ON THE LIMIT CYCLES SURROUNDING A DIAGONALIZABLE LINEAR NODE WITH HOMOGENEOUS NONLINEARITIES

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Abstract. In this paper we study the existence and non–existence of limit cycles for the class of polynomial differential systems of the form

$$\dot{x} = \lambda x + P_n(x, y), \qquad \dot{y} = \mu y + Q_n(x, y),$$

where P_n and Q_n are homogeneous polynomials of degree n.

1. Introduction and statement of the main results

A *polynomial differential system* in $\mathbb{R}^2$ is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \qquad \frac{dy}{dt} = \dot{y} = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are polynomials in the variables x and y with real coefficients. Then $m = \max\{\deg P, \deg Q\}$ is the *degree* of the polynomial system.

As usual a *limit cycle* of a system (1) is an isolated periodic solution in the set of all periodic solutions of system (1). Limit cycles of planar differential systems were defined by Poincaré [21] and started to be studied intensively at the end of the 1920s by van der Pol [22], Liénard [12] and Andronov [1].

In the qualitative theory of the polynomial differential equations in the plane $\mathbb{R}^2$ one of the more difficult problems is the study of their limit cycles. Thus the second part of the unsolved 16–th Hilbert problem [13] asked for an upper bound on the maximum number of limit cycles for the polynomial differential systems of a given degree in function of this degree, see for more details the surveys [14] and [11].

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