PHASE PORTRAITS OF THE SELKOV MODEL IN THE POINCARÉ DISC

JAUME LLIBRE[1] AND AREFEH NABAVI[2]

Abstract. In this paper we classify the phase portraits in the Poincaré disc of the Selkov model

$$\dot{x} = -x + ay + x^2y, \quad \dot{y} = b - ay - x^2y,$$

in function of its parameters $a, b \in \mathbb{R}$. We determine the regions in the parameter plane with biological meaning.

1. Introduction and statement of the main results

In this paper we consider the Selkov model of glycolisis which is given by the following cubic differential system

$$\dot{x} = -x + ay + x^2y, \quad \dot{y} = b - ay - x^2y, \tag{1}$$

where x and y are the concentrations, and a and b are kinetic parameters. As usual the dot denotes derivative with respect to the time t. For more biological meanings about this system see the papers [10, 11], and the references quoted there. For other related models see for instance [3].

Our purpose is to classify the global phase portraits of system (1) in the Poincaré disc, in function of its parameters, and determine the regions of the parameters with biological meaning.

Roughly speaking the Poincaré disc is the closed disc $\mathbb{D}^2$ centered at the origin of $\mathbb{R}^2$ of radius one, its interior is identified with $\mathbb{R}^2$ and its boundary, the circle $\mathbb{S}^1$ is identified with the infinity of $\mathbb{R}^2$, because in $\mathbb{R}^2$ we can go to the infinity in many directions as points has the circle $\mathbb{S}^1$. Then polynomial vector fields $\mathbf{X}$, as the ones defined by system (1), can be extended to analytic vector fields $p(\mathbf{X})$ defined in

2010 *Mathematics Subject Classification.* Primary 34C05. Secondary 34C07, 37C08.

Key words and phrases. Selkov model, Phase portrait, Poincaré compactification.