ON THE GLOBAL DYNAMICS OF A THREE-DIMENSIONAL FORCED-DAMPED DIFFERENTIAL SYSTEM

JAUME LLIBRE[1], Y. PAULINA MARTÍNEZ[2] AND CLÀUDIA VALLS[3]

Abstract. In this paper by using the Poincaré compactification of $\mathbb{R}^3$ we make a global analysis of the model $x' = -ax+y+yz$, $y' = x-ay+bxz$, $z' = cz - bxy$. In particular we give the complete description of its dynamics on the infinity sphere. For $a+c=0$ or $b=1$ this system has invariants. For these values of the parameters we provide the global phase portrait of the system in the Poincaré ball. We also describe the α and ω-limit sets of its orbits in the Poincaré ball.

1. Introduction and statement of the main results

We consider the autonomous polynomial differential system

$$\begin{aligned} \dot{x} &= -ax + y + yz, \\ \dot{y} &= x - ay + bxz, \\ \dot{z} &= cz - bxy, \end{aligned} \tag{1}$$

where a, b, c are real parameters and $b > 0$. As usual the dot denotes derivative with respect to the time t. This system was proposed and studied by Pehlivan [9] extending a previous study of Craik and Okamoto [2] including linear forcing and damping. It is a relevant system because it arises in mechanical, electrical and fluid-mechanics (see for instance [7, 8] and the references therein) and can display simultaneously unbounded and chaotic solutions, a non common phenomena in chaotic systems. In this way some important properties are similar to the properties of the well-known Lorenz system (see [6] and the references therein). For instance one can easily check that system (1) is invariant under the change of variables $(x, y, z) \mapsto (-x, -y, z)$ consequently if $(x(t), y(t), z(t))$ is a solution of system (1), then $(-x(t), -y(t), z(t))$, i.e. its symmetric with respect to the z-axis, is also a solution. Moreover, since the divergence of system (1) is $c - 2a$ the phase volume under the flow of system (1) shrinks uniformly if $c - 2a < 0$ (as it happens in the Lorenz system for certain relation on its parameters). Thus in this case the attractor presented by this system has zero Lebesgue

2010 *Mathematics Subject Classification.* 34C05, 34C07, 34C08.

Key words and phrases. Global dynamics, Poincaré compactification, forced-damped system, invariant algebraic curve, invariant.