ON THE 16-TH HILBERT PROBLEM FOR DISCONTINUOUS PIECEWISE POLYNOMIAL HAMILTONIAN SYSTEMS

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Abstract. In this paper we study the maximum number of limit cycles of the discontinuous piecewise differential systems with two zones separated by the straight line $y = 0$, in $y \geq 0$ there is a polynomial Hamiltonian system of degree m, and in $y \leq 0$ there is a polynomial Hamiltonian system of degree n.

First for this class of discontinuous piecewise polynomial Hamiltonian systems, which are perturbation of a linear center, we provide a sharp upper bound for the maximum number of the limit cycles that can bifurcate from the periodic orbits of the linear center using the averaging theory up to any order.

After for the general discontinuous piecewise polynomial Hamiltonian systems we also give an upper bound for their maximum number of limit cycles in function of m and n. Moreover, this upper bound is reached for some degrees of m and n.

1. Introduction and statement of main results

One of the most studied problems in the qualitatitve theory of the differential equations in the plane is to identify the maximum number of limit cycles that can exhibit a given class of differential systems. Thus a famous and challenging question is the Hilbert's 16th problem [22], which was proposed in 1900. In the second part of this question, Hilbert asked what is the maximum number of limit cycles that planar polynomial differential system of a given degree may have. Since 1900 many researchers are dedicated to study this problem and some excellent results were obtained, see for instance the survey paper [26]. But this question is far from being answered up to now, even for quadratic polynomial differential systems. Let $H(n)$ be the maximum number of limit cycles that a planar polynomial differential system of degree n may have. Sometimes, $H(n)$ is called *Hilbert number*. As far as we are concerned, the existing results showed $H(2) \geq 4, H(3) \geq 13, H(4) \geq 21, H(5) \geq 33, H(6) \geq 44, H(7) \geq 65, H(8) \geq 76$, etc, see [15, 17, 20, 27, 19, 33, 37, 39]. For n sufficiently large it is known that

$$H(n) \geq \frac{(n+2)^2 \ln(n+2)}{2 \ln 2},$$

see [10, 26, 19].

In these last twenty years an increasing interest has appeared for studying the discontinuous piecewise smooth differential systems, stimulated by lots of nonsmooth or discontinuous phenomena that come from mechanical engineering with dry frictions, feedback control systems, electrical circuits with switches, neuron models, biology, see for example

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