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Dynamics and Mission Design Near Libration Points

Vol. II Fundamentals: The Case of Triangular Libration Points

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Vol. II Fundamentals: The Case of Triangular Libration Points

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Preface

It is well-known that the restricted three-body problem has triangular equilibrium points. Those points are linearly stable for values of the mass parameter, μ, below the Routh's critical value μ_1. It is also known that in the spatial case it is nonlinearly stable, not for all the initial conditions in a neighborhood of the equilibrium points L_4, L_5 but for a set of relatively big measure. The fraction of stable motions tends to 1 when the size of the neighborhood tends to 0, for almost all $\mu \in (0, \mu_1)$. This follows from the celebrated Kolmogorov–Arnold–Moser theorem. In fact there are neighborhoods of computable size for which one obtains "practical stability" in the sense that the massless particle remains close to the equilibrium point for a big time interval (some millions of years, say).

The question is which part of this stability subsists when the idealized RTBP is substituted by the Earth–Moon system with its real motion and under the very strong influence of the Sun and the milder perturbations due to planets, solar radiation pressure, no spherical shape of the Earth and Moon, etc.

According to the literature, what has been done in the problem follows two approaches:

a) Numerical simulations of more or less accurate models of the real solar system. Usually the starting point is taken at one of the equilibrium points L_4, L_5. The results are slightly confusing. Depending on the initial epoch chosen, the orbit escapes in a few months or behaves according to the pattern that we proceed to describe. First the particle spirals from the equilibrium point outwards until it reaches a size of the order of magnitude of the Earth–Moon distance. Then the particle spirals inwards going close again to the equilibrium point. The behavior repeats itself several times or, eventually, escapes after some of these big oscillations, when a closed encounter with one of the primaries is produced.

b) Study of periodic or quasi-periodic orbits of some much simpler problem. This can be the bicircular model or a coherent system close to the bicircular one and still periodic or a Hamiltonian system retaining a few leading terms of the equations. In this case, the methods of perturbation theory, mainly those based on Lie series, lead to much simpler auxiliary Hamiltonians that can be studied analytically. The other cases can be studied in turn numerically or semianalytically. The results are again confusing: small changes in the approach produce big changes in the size of the periodic orbits or they even disappear. This is a consequence of the lack of convergence of the methods used and of the sensitivity to some resonances.

The concrete questions that are studied in this book are:

a) Is there some orbit of the real solar system which looks like the periodic orbits of the previous item b) ? That is, are there orbits performing revolutions around L_4 covering, eventually a thick strip ? Furthermore we would be pleased if those orbits be quasi-periodic, at least if the motion of the bodies of the solar system is assumed to be quasi-periodic with respect to time. The present knowledge of the motion of the main bodies of the solar system ensures that this assumption can be accepted for moderate time intervals, much larger than the ones for present planned missions. However there is no guarantee that the orbits we look for exist nor they be quasi-periodic.
b) If the orbit of a) exists and two particles (spacecrafts) are put close to it, how does the mutual distance and orientation change with time ?

As a final conclusion of the work, there is evidence that orbits moving in a somewhat big annulus around L_4 and L_5 exist, that these orbits have small components out of the plane of the Earth–Moon system, and that they are at most mildly unstable.

The mutual distance of two points starting close to these orbits changes by an important factor (at most 1 to 100), and the orientation changes in a regular way, unless some small loops are present in the projection of the relative motion on the (x, y)-plane or this projection comes too close to the origin.

In any case we believe that it can be a useful place to locate one or two spacecrafts for scientific purposes because of the nice properties concerning stability. The station keeping necessary to maintain the orbit in its right place can be reduced to an unimportant amount.

The contents of this book is the final report of the study contract that was done for the European Space Agency in 1987. This report is reproduced textually with minor modifications: the detected typing or obvious mistakes have been corrected, some tables have been shortened and references, which appeared as preprints in the report, have been updated. The layout of the (scanned) figures has changed slightly, to accommodate to latex requirements.

The last page of this preface reproduces the cover page of the report for the European Space Agency showing, in particular, the original title of the study.

For the ESA's study we also produced software that is not included here, although all its main modules are described in detail in the text.

Updates on the state of the art, both concerning theoretical and practical studies, can be found at the end of Volume IV of this collection of works on *Dynamics and Mission Design Near Libration Points.*

STUDY ON ORBITS NEAR THE TRIANGULAR LIBRATION POINTS IN THE PERTURBED RESTRICTED THREE-BODY PROBLEM

FINAL REPORT

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