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Generalized Rings Around the McMullen Domain

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Abstract
We consider the family of rational maps given by $F_\lambda(z) = z^n + \lambda/z^d$ where $n, d \in \mathbb{N}$ with $1/n + 1/d < 1$, the variable $z \in \widehat{\mathbb{C}}$ and the parameter $\lambda \in \mathbb{C}$. It is known that when $n = d \geq 3$ there are infinitely many rings $\mathcal{S}^k$ with $k \in \mathbb{N}$, around the McMullen domain. The McMullen domain is a region centered at the origin in the parameter λ-plane where the Julia sets of F_λ are Cantor sets of simple closed curves. The rings $\mathcal{S}^k$ converge to the boundary of the McMullen domain as $k \to \infty$ and contain parameter values that lie at the center of Sierpiński holes, i.e., open simply connected subsets of the parameter space for which the Julia sets of F_λ are Sierpiński curves. The rings also contain the same number of superstable parameter values, i.e., parameter values for which one of the critical points is periodic and correspond to the centers of the main cardioids of copies of Mandelbrot sets. In this paper we generalize the existence of these rings to the case when $1/n + 1/d < 1$ where n is not necessarily equal to d. The number of Sierpiński holes and superstable parameters on $\mathcal{S}^1$ is $\tau_1^{n,d} = n - 1$, and on $\mathcal{S}^k$ for $k > 1$ is given by $\tau_k^{n,d} = dn^{k-2}(n-1) - n^{k-1} + 1$.

Keywords Singularly perturbed rational maps · McMullen domain · Baby Mandelbrot sets · Sierpinski holes

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