

LIMIT CYCLES FROM A MONODROMIC INFINITY IN PLANAR PIECEWISE LINEAR SYSTEMS

EMILIO FREIRE, ENRIQUE PONCE, JOAN TORREGROSA, AND FRANCISCO TORRES

ABSTRACT. Planar piecewise linear systems with two linearity zones separated by a straight line and with a periodic orbit at infinity are considered. By using some changes of variables and parameters, a reduced canonical form with five parameters is obtained. Instead of the usual Bendixson transformation to work near infinity, a more direct approach is introduced by taking suitable coordinates for the crossing points of the possible periodic orbits with the separation straight line. The required computations to characterize the stability and bifurcations of the periodic orbit at infinity are much easier. It is shown that the Hopf bifurcation at infinity can have degeneracies of co-dimension three and, in particular, up to three limit cycles can bifurcate from the periodic orbit at infinity. This provides a new mechanism to explain the claimed maximum number of limit cycles in this family of systems. The centers at infinity classification together with the limit cycles bifurcating from them are also analyzed.

1. INTRODUCTION AND MAIN RESULTS

The analysis of piecewise linear systems is nowadays an active field of research since certain modern devices are well-modeled by this class of systems, see [10]. Even for the simplest situation, as is the case of the aggregation of two planar linear systems, there are still unsolved problems; for instance, it is known that such discontinuous piecewise linear systems can have three limit cycles (see, for instance [4, 16, 20, 24, 25]) but we still do not know if three is indeed the maximum number for them.

In the analysis of the dynamical richness in a differential system, an interesting source of knowledge comes from the study of all possible bifurcations that the system can undergo. Furthermore, it should be emphasized the relevance of including in such a bifurcation study the possible bifurcations from infinity. Here, we explore the information on the maximum number of limit cycles that can be obtained by studying the periodic orbit at infinity and its possible bifurcations in the mentioned family of planar discontinuous piecewise linear systems with two zones separated by a straight line. Bifurcations from infinity for planar piecewise linear differential systems has been analyzed before in [23], and more recently in [19]. In [23] only continuous cases with two zones and three zones with symmetry were considered. For the two-zones case, only one bifurcating limit cycle was detected, according to the well-known fact that there can be only one limit cycle in the class of continuous planar piecewise linear differential systems with two zones separated by a straight line, see [12]. In [19], the bifurcation from infinity is addressed for the case of discontinuous piecewise linear differential systems, by perturbing in a non-symmetric

2010 *Mathematics Subject Classification.* Primary: 37G15, 34C07; Secondary: 34C25, 34C23.

Key words and phrases. Planar piecewise linear systems; Bifurcation from infinity; Limit cycles; Centers.