Limit cycles on piecewise smooth vector fields with coupled rigid centers

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Abstract We provide an upper bound for the maximum number of limit cycles of classes of discontinuous piecewise differential systems formed by two differential systems separated by a straight line, and assuming that each one of these differential systems has a rigid center and are composed by a linear part and a homogeneous polynomial nonlinear part. For these kind of discontinuous piecewise differential systems we solve the 16th Hilbert problem.

Keywords Piecewise smooth vector fields · rigid centers · limit cycles

1 Introduction and main results

The search for limit cycles is one of the most important studies in the qualitative theory of the planar ordinary differential equations. Such importance is evidenced by the 16th Hilbert's problem (see [18]): the determination of an upper bound for the number of limit cycles for the class of planar polynomial vector fields of degree n. This problem remains unsolved for $n \geq 2$.

In the last decades the study of discontinuous piecewise vector fields has had a great growth in the mathematical community, since such vector fields can be used as important models in applied science. Indeed, several models used in applied problems are described by systems that are not completely differentiable, but in different parts, where a law of evolution is suddenly interrupted by another law of evolution that will begin to govern such system. The modeling of such systems consists of different vector fields defined in distinct regions separated by a switching manifold and are known as piecewise smooth vector fields, discontinuous piecewise systems, or Filippov systems.

Pioneering studies initiated by Andronov [3] and Filippov [13] led to a theoretical foundation for this kind of differential systems. Nowadays a vast literature on these vector fields is available. See for instance [6] for the main theory and some applications, [10] for applications in electrical circuits, [8, 22] for applications in mechanical models, [7, 21] for applications in relay systems, among others. As in the regular case the study of the existence and location of limit cycles in piecewise smooth vector fields is also of great importance, because in addition to the smooth dynamic elements, there are new ones which did not exist in the smooth world.

Let $p \in \mathbb{R}^2$ be a singularity of an analytic differential system in the plane. The singularity p is a *center* if there exists an open neighborhood U of p such that all the solutions in $U \setminus \{p\}$ are periodic. Without loss of generality we may assume that the equilibrium point is at the origin. Denote by $\mathcal{T}_q$ the period of the periodic orbit through $q \in U \setminus \{p\}$. We say that p is an *isochronous center* if $\mathcal{T}_q$ is constant for all $q \in U \setminus \{p\}$. An isochronous center is *uniform* or *rigid* if the angular velocity of the vector field is the same for all periodic orbits in $U \setminus \{p\}$, that is if in polar coordinates (r, θ) defined by $x = r\cos\theta$, $y = r\sin\theta$ it can be written as $\dot{r} = G(r, \theta)$, $\dot{\theta} = k$, where $k \neq 0$

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