Zero-Hopf bifurcation in the general Van der Pol-Duffing equation

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Abstract

We investigate the dynamical behavior of the general Van der Pol-Duffing equation $\dot{x} = -\nu\left(x^3 - \mu x - y\right)$, $\dot{y} = -hz + kx - \alpha y$, $\dot{z} = \beta y$. This model generalizes the behavior of a Van der Pol-Duffing circuit with parallel resistor. We prove the existence of periodic solutions that bifurcate from the origin of coordinates for different values of the parameters. For some of these values, we can prove the existence of up to three periodic solutions and also two invariant tori that may bifurcate from the origin of that system. These results are obtained using very recent results on averaging theory.

Keywords: Averaging theory, Periodic solutions, Zero–Hopf bifurcation,
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1. General Van der Pol-Duffing Equation

Introduction and statements of the main results

The aim of this paper is to study the bifurcation of periodic solutions and invariant tori in the general Van der Pol-Duffing differential system

$$\begin{aligned} \dot{x} =& -\nu\left(x^3 - \mu x - y\right), \\ \dot{y} =& -hz + kx - \alpha y \\ \dot{z} =& \beta y, \end{aligned} \tag{1}$$

where α, h, β, k, ν and μ are real parameters. In 2008, Matouk and Agiza [5] studied the Hopf bifurcation in system (1) with $\alpha = h = -1$. They also

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