Journal of Computational and Applied Mathematics 368 (2020) 112456

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

The centers and their cyclicity for a class of polynomial differential systems of degree 7

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ARTICLE INFO

Article history:
Received 15 June 2018
Received in revised form 30 July 2019

MSC:
primary 34C15
34C25

Keywords:
Center
Phase portrait
Cyclicity
Limit cycle
Hopf bifurcation
Averaging method

ABSTRACT

We classify the global phase portraits in the Poincaré disc of the generalized Kukles systems

$$\dot{x} = -y, \qquad \dot{y} = x + axy^6 + bx^3y^4 + cx^5y^2 + dx^7,$$

which are symmetric with respect to both axes of coordinates. Moreover using the averaging theory up to sixth order, we study the cyclicity of the center located at the origin of coordinates, i.e. how many limit cycles can bifurcate from the origin of coordinates of the previous differential system when we perturb it inside the class of all polynomial differential systems of degree 7.

1. Introduction and statement of the main results

Two classical and difficult problems of the qualitative theory of planar polynomial differential systems are the characterization of their centers, and the study of their cyclicity, i.e. how many limit cycles can bifurcate from a center when we perturb it inside a given class of polynomial differential systems. Of course, this kind of bifurcation is called in the literature a Hopf bifurcation.

In this work we deal with planar polynomial differential systems of the form

$$\dot{x} = -y, \quad \dot{y} = x + Q_n(x, y), \tag{1}$$

having a center at the origin, being $Q_n(x, y)$ a homogeneous polynomial of degree n. As usual the dot in system (1) denotes derivative with respect an independent variable t usually called the time. Systems of this form were called by Giné [1] *Kukles homogeneous systems*.

In 1999 Volokitin and Ivanov [2] conjectured that the systems (1) have a center at the origin if and only if they are symmetric with respect to one of the coordinate axes. For $n = 2$ and $n = 3$, the authors of the conjecture knew that it holds. Giné [1] in 2002 proved the conjecture for $n = 4$ and $n = 5$. Giné et al. [3,4] proved the conjecture for all n under an additional assumption, that the authors believe that it is redundant.

In this work we consider the class of polynomial differential systems (1) for $n = 7$ which are symmetric with respect to both coordinate axes, i.e.

$$\dot{x} = -y, \quad \dot{y} = x + axy^6 + bx^3y^4 + cx^5y^2 + dx^7. \tag{2}$$

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https://doi.org/10.1016/j.cam.2019.112456
0377-0427/