GLOBAL TOPOLOGICAL CONFIGURATIONS OF SINGULARITIES FOR THE WHOLE FAMILY OF QUADRATIC DIFFERENTIAL SYSTEMS

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Abstract. In [1] the authors proved that there are 1765 different global geometrical configurations of singularities of quadratic differential systems in the plane. There are other 8 configurations conjectured impossible, all of them related with a single configuration of finite singularities. This classification is completely algebraic and done in terms of invariant polynomials and it is finer than the classification of quadratic systems according to the topological classification of the global configurations of singularities, the goal of this article. The long term project is the classification of phase portraits of all quadratic systems under topological equivalence. A first step in this direction is to obtain the classification of quadratic systems under topological equivalence of local phase portraits around singularities. In this paper we extract the local topological information around all singularities from the 1765 geometric equivalence classes. We prove that there are exactly 208 topologically distinct global topological configurations of singularities for the whole quadratic class. The 8 global geometrical configurations conjectured impossible do not affect this number of 208. From here the next goal would be to obtain a bound for the number of possible different phase portraits, modulo limit cycles.

1. Introduction and statement of main results

We consider here differential systems of the form

$$\frac{dx}{dt} = p(x, y), \qquad \frac{dy}{dt} = q(x, y), \tag{1}$$

where $p,\, q \in \mathbb{R}[x, y]$, i.e. p, q are polynomials in x, y over $\mathbb{R}$. We call *degree* of a system (1) the integer $m = \max(\deg p, \deg q)$. In particular we call *quadratic* a differential system (1) with $m = 2$. We denote here by **QS** the whole class of real quadratic differential systems.

Polynomial systems (1) intervene in many areas of applied mathematics. They are also interesting from the theoretical viewpoint since some problems stated over 100 years ago are still unsolved even for the quadratic class. Hilbert's 16th problem asks to determine for each positive integer n the maximum number of limit cycle which a system (1) of degree n could have, in case there is a finite bound for the number of limit cycles of such systems. So far, not even the finiteness part of Hilbert's 16'th problem was proved and this not even for quadratic systems. The finiteness part of Hilbert's 16th problem asks for a proof that for every positive integer n there exists an integer N such that every system (1) of degree n has at most N limit cycles. These are not the only longstanding open problems on systems (1).

There is a large number of papers on quadratic systems. For early short surveys on quadratic systems see [5, 4]. A more recent account on quadratic systems is given in [9] which contains many phase portraits of quadratic systems, the harder cases being left open. The most recent and most complete survey on quadratic systems is given in the book [1] where the family **QS** is classified according to the geometric equivalence relation for configurations of singularities of the systems. In this book it was proved that there are at least 1765 and at most 1773 such geometric configurations of singularities for quadratic differential systems.

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