

ON THE PERIODIC ORBIT BIFURCATING FROM A HOPF BIFURCATION IN SYSTEMS WITH TWO SLOW AND ONE FAST VARIABLES

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ABSTRACT. The Hopf bifurcation in slow-fast systems with two slow variables and one fast variable has been studied recently, mainly from a numerical point of view. Our goal is to provide an analytic proof of the existence of the zero Hopf bifurcation exhibited for such systems, and to characterize the stability or instability of the periodic orbit which borns in such zero Hopf bifurcation. Our proofs use the averaging theory.

1. INTRODUCTION AND STATEMENT OF THE RESULTS

Hopf bifurcations have been studied intensively in two dimensional differential systems with one slow and one fast variable, see for instance [4, 6, 7, 11, 8].

Canard explosions are associated with these singular Hopf bifurcations, manifested by a very rapid growth in the amplitude of periodic orbits, see for instance [2, 3, 5, 12].

There has been less analysis of the Hopf bifurcations in slow-fast systems with two slow variables and one fast variable, see for instance [9, 13, 16].

In this paper we shall study this last kind of Hopf bifurcation using the averaging theory and in particular we shall characterize the stability or instability of the periodic orbit which bifurcates in the Hopf bifurcation. We shall follow the work of Guckenheimer [9] who reduces the study of the mentioned Hopf bifurcation to study the zero Hopf bifurcation of the differential system

$$(1) \quad X' = Y - X^2, \quad Y' = Z - X, \quad Z' = -\mu - AX - BY - CZ,$$

where the prime denotes derivative with respect to τ . A summary of this reduction process is done in appendix I, for more details see [9].

We note that when

$$(2) \quad B \neq 0 \quad \text{and} \quad (A + C)^2 - 4B\mu \geq 0,$$

the differential system (1) possesses exactly one equilibrium point with coordinates (X^*, X^{*2}, X^*) where

$$X^* = -\frac{A + C - \sqrt{(A + C)^2 - 4B\mu}}{2B}.$$

The *zero Hopf bifurcation* (also called *saddle-node Hopf bifurcation* or *fold Hopf*) occurs at the equilibrium point (X^*, X^{*2}, X^*) when it has one zero eigenvalue and

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