HOPF BIFURCATION IN THE FULL REPRESSILATOR EQUATIONS

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Abstract. In this paper we prove that the full repressilator equations, in dimension six undergo a supercritical Hopf bifurcation.

1. Introduction

Oscillatory networks is a particular kind of regulatory molecular networks, i.e., collections of interacting molecules in a cell. The regulatory oscillators can be used to study abnormalities of process in the cell, from sleep disorders to cancer. So, they attract significant attention among biologists and biophysicists. There are many implementations of artificial oscillatory networks (see, e.g., [1, 2, 6, 7, 8, 10, 11]). One of them is the repressilator [5]. Its genetic implementation uses three proteins that cyclically repress the synthesis of one another. The following system of differential equations describes the behavior of the repressilator:

$$
\begin{aligned}
\dot{m}_1 &= -m_1 + \frac{\alpha}{1+v^n} + \alpha_0,\\
\dot{m}_2 &= -m_2 + \frac{\alpha}{1+w^n} + \alpha_0,\\
\dot{m}_3 &= -m_3 + \frac{\alpha}{1+u^n} + \alpha_0,\\
\dot{u} &= -\beta(u - m_1),\\
\dot{v} &= -\beta(v - m_2),\\
\dot{w} &= -\beta(w - m_3).
\end{aligned}
\tag{1}
$$

Here u, v and w are proportional to the protein concentration, while m_i are proportional to the concentration of mRNA corresponding to those proteins. The nonlinear function $f(x) = \dfrac{\alpha}{1+x^n}$ reflects synthesis of the mRNAs from the DNA controlled by regulatory elements, and n is the cooperatively. The parameter α_0 represents uncontrolled part of the mRNA synthesis, and it is

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